

STABILITY ANALYSIS OF SINGLE-WAVELENGTH OPTICAL BUFFERS

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Optical Burst Switching provides a future-proof alternative to the current electronic switching in the backbone, but has buffering implemented with a set of Fiber Delay Lines (FDL). The resulting buffering system fundamentally differs from a classic one because the set of possible waiting times (unlike the classic case) is a numerable set, each value corresponding to the length of a delay line. We present the stability conditions of the optical buffer system using a regenerative approach in the spirit of [13].

Keywords: stochastic processes, optical buffers, stability, regeneration.

1. OPTICAL BUFFER SETTING

The optical buffer system is characterized by the length of the FDLs, a denumerable set $\mathcal{A} = \{a_0, a_1, a_2, \dots\}$ of available delays $a_i \in \mathbb{R}^+$, $i \in \mathbb{N}$, where $a_0 = 0$.

As the set of lines are intended to resolve contention, it is necessary that contending bursts undergo different delays, and therefore, a useful FDL set never contains the same length twice, $a_i \neq a_j$ for $i \neq j$. Also, we sort the line lengths in ascending order, $a_0 < a_1 < \dots$ assuming $\lim_{i \rightarrow \infty} a_i = \infty$. The latter ensures that a suitable delay line can always be found.

The main characteristic of an optical buffer is that it cannot always assign the exact delay value needed. Therefore, when a delay x is requested, the actual delay a_i is chosen from the FDL set \mathcal{A} such that $a_{i-1} < x \leq a_i$. To keep FIFO scheduling discipline, the resulting assignment procedure (select a_i given x) is as

$$\lceil x \rceil_{\mathcal{A}} = \inf\{a_i \in \mathcal{A} : a_i \geq x\}, \quad x \in \mathbb{R}^+. \quad (1)$$

State-of-art and more details on optical buffer systems can be found in [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16].

We consider a single-server optical buffer system with renewal input, and let t_k be the arrival instant of burst $k \geq 1$. Denote the i.i.d. interarrival times $T_k = t_{k+1} - t_k$, and let B_k be the service time of burst k , $k \geq 1$. Also introduce the iid series $U_k =$

$B_k - T_k$, $k \geq 1$. In the optical system, the waiting time sequence $W = \{W_k\}$ satisfies the recursion

$$W_{k+1} = \lceil W_k + U_k \rceil_{\mathcal{A}}, \quad k \geq 1. \quad (2)$$

2. REGENERATIVE STABILITY ANALYSIS

We introduce notations

$$g_n = a_{n+1} - a_n, \quad n \geq 0; \quad \Delta^* = \sup_{n \geq 0} g_n; \quad \Delta_0 = \limsup_{n \rightarrow \infty} g_n; \quad \delta^* = \inf_{n \geq 0} g_n. \quad (3)$$

Also denote $U = B - T$, where B, T are generic variables for B_k and T_k , respectively. We construct classical regenerations $\{\beta_n, n \geq 1\}$ for the process W by the following (conventional) way: let $\beta_0 = 0$ and

$$\beta_{n+1} = \inf(k > \beta_n : W_k = 0), \quad n \geq 0 \quad (\inf \emptyset = \infty). \quad (4)$$

Our purpose is to establish conditions which imply positive recurrence of the process β , that is

$$\beta_1 < \infty \text{ with probability } 1 \text{ (w.p.1) and } \mathbf{E}(\beta_2 - \beta_1) \equiv \alpha < \infty. \quad (5)$$

In the zero-delayed case, all regeneration periods are stochastically equivalent, that is $\beta_1 =_{st} \beta_2 - \beta_1$. In other words, the process W starts at regeneration instant, $W_1 = 0$ and $\mathbf{E}\beta_1 = \alpha$. Note that the process $\beta = \{\beta_n\}$ is well-defined since $a_0 = 0 \in \mathcal{A}$.

In the proof we use characterization (7) of the recurrence property of the renewal process β via the limiting behavior of the forward regeneration time at instant n , which is defined as

$$\beta(n) = \inf_k (\beta_k - n : \beta_k - n > 0), \quad n \geq 1. \quad (6)$$

It is known [6] that

$$\alpha = \infty \text{ if and only if } \beta(n) \rightarrow \infty \text{ in probability as } n \rightarrow \infty. \quad (7)$$

Note that if $\mathbf{E}\beta_1 = \alpha$, then $\beta_1 < \infty$ w.p.1 provided $\alpha < \infty$. It then follows that positive recurrence holds if we show that $\alpha < \infty$.

We assume that

$$\mathbf{E}B < \infty, \quad \mathbf{E}T < \infty, \quad (8)$$

that

$$\delta^* > 0, \quad \Delta^* < \infty, \quad (9)$$

that the following *negative drift* condition

$$\Delta_0 + \mathbf{E}U < 0 \quad (10)$$

holds, and also that the following *regeneration* assumption

$$\mathbf{P}(T > \Delta^* + B) > 0 \quad (11)$$

holds. The main stability result is the following.

Theorem 1. *Under assumptions (8-11), the zero-delayed renewal process β satisfying (4) is positive recurrent, that is (5) holds.*

The proof consist of two main steps. First we shows that the given conditions imply the basic workload process has a negative drift outside a compact set. Then it follows that the waiting time process sequence visits the compact set with a positive probability infinitely often. This then implies the second step where we show that, starting within the compact set, the process hits a regeneration instant within a finite interval with a probability which is (uniformly) lower bounded over the set by a positive constant. It means that the forward regeneration time does not go to infinity as time increases and immediately implies positive recurrence by (5).

Also we present an additional condition which (together with assumptions (8-11)) implies positive recurrence of the *delayed* process β under any initial value β_1 .

We note that indeed the system is state-dependent which makes its stability analysis more difficult than in a classical case.

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